1a $F = I h B$

where I is the induced current and h is the height of the rectangle

$$|\mathcal{E}| = \left| -\frac{d\phi_B}{dt} \right| = \frac{d\phi_B}{dt} = h v B$$

Using law of Ohm $\mathcal{E} = IR$
and resistance $R = \frac{\rho L}{A} = \frac{\rho L}{\pi r^2}$

we find:

$$F = \frac{\mathcal{E}}{R} h B = \frac{h v B \pi r^2}{\rho L} h B$$

$$= \frac{h^2 B^2 \pi r^2 v}{\rho L}$$

1b If we double the force keeping everything else the same, v will be twice as large. The time will be reduced by a factor of two $t = 0.5$ s.

1c If the resistivity doubles, the force is halved so again you will have 1 N.

1d If the radius doubles from 5 mm to 10 mm, the force changes by a factor of 4; $F = 4$ N.

2 a. $\hat{n}$ has direction of $\vec{E}$

This is $\hat{y}$; polarization is $\hat{y}$.

b argument of sin is $kx+\omega t$

therefore wave proceeds in $-x$ direction

c. $$\vec{\nabla} \cdot \vec{E} = \frac{\partial E_x}{\partial x} + \frac{\partial E_y}{\partial y} + \frac{\partial E_z}{\partial z} = 0 ;$$ OK no charges

same for $\vec{\nabla} \cdot \vec{B} = 0$ as well ; OK no charges

$$\vec{\nabla} \times \vec{E} = k E_0 \cos(kx + \omega t) \hat{z}$$

$$\vec{\nabla} \times \vec{B} = k_0 \frac{E_0}{c} \cos(kx + \omega t) \hat{y}$$

$$-\frac{\partial \vec{B}}{\partial t} = \omega \frac{E_0}{c} \cos(kx+\omega t) \hat{z}$$

$$\frac{1}{\mu_0 \varepsilon_0} \frac{\partial \vec{E}}{\partial t} = + \frac{1}{\mu_0 \varepsilon_0} \omega E_0 \cos(kx+\omega t) \hat{y}$$

d Satisfy M's equation : $k = \frac{\omega}{c}$

d $$k = \frac{2\pi}{\lambda} = \frac{\omega}{c} \qquad \lambda = \frac{2\pi c}{\omega} = \frac{2\pi \, 3 \, 10^8}{10^{10}} = 18 \text{ cm}$$

e $$u = \frac{1}{2}\left(\varepsilon_0 E^2 + \frac{1}{\mu_0} B^2\right) \qquad \boxed{\frac{\partial u}{\partial t} + \vec{\nabla} \cdot \vec{S} = 0}$$

$$= \frac{1}{2} \cancel{\frac{1}{2} 2} \varepsilon_0 E_0^2 = \frac{1}{2} \varepsilon_0 E_0^2 = \frac{1}{2}\left(\frac{1}{2}\right)^2 8.9 \, 10^{-12}$$

$$\simeq 10^{-12} \text{ J/m}^3$$

3 The current I charges the caps of the fat wire ends.

$$\frac{\partial E}{\partial t} = \frac{1}{\varepsilon_0}\frac{\partial \sigma}{\partial t} = \frac{1}{\varepsilon_0}\frac{\partial Q}{A\,\partial t} = \frac{1}{\varepsilon_0 A} I.$$

with $A = \pi a^2$.

$$\vec{\nabla} \times \vec{B} = \mu_0 \vec{J}_{free} + \mu_0 \varepsilon_0 \frac{\partial \vec{E}}{\partial t}$$

$$J_{free} = 0 \qquad \varepsilon_0 \frac{\partial \vec{E}}{\partial t} = \frac{I}{A}.$$

For an amperian loop with radius s we find

$$\oint \vec{B}\cdot d\vec{l} = \mu_0 I_{encl} = \mu_0 I \frac{s^2}{a^2}$$

$$B\, 2\pi s = \mu_0 I \frac{s^2}{a^2}$$

$$B = \frac{\mu_0 I s}{2\pi a^2}.$$

2.l Alternative $\langle u \rangle = \frac{1}{2}\varepsilon_0 E_0^2$ for an electromagnetic wave in vacuum.

$I = \langle S \rangle$ = time averaged poynting vector

$$\langle u \rangle = \frac{\langle S \rangle}{c}$$

$$\langle S \rangle = \frac{1}{2} c\, \varepsilon_0 E_0^2 = \frac{1}{2} \cdot 3 \cdot 10^8 \cdot 8{,}85 \cdot 10^{-12} (0{,}5)^2$$

$$3.3 \cdot 10^{-4}\ J/(s m^2)$$

$$\langle u \rangle = \frac{\langle S \rangle}{c} = 1.1 \cdot 10^{-12} J/m^3$$

4 a. $\vec{E} = \frac{1}{4\pi\varepsilon_0} \frac{q}{s^2} \hat{s}$

Projection on to the xy plane gives:

$E_{plane,1} = \frac{1}{4\pi\varepsilon_0} \frac{q_1}{s^2} \cos\theta$ for each charge

Total E_{plane} is therefore: $E_{plane,1,2} = \frac{2}{4\pi\varepsilon_0} \frac{q}{s^2} \cos\theta$

$s^2 = a^2 + r^2$

$\cos\theta = \frac{r}{s} = \frac{r}{(a^2+r^2)^{1/2}}$

$E_x = \cos\varphi \; E_{plane,1,2} = \frac{2}{4\pi\varepsilon_0} \frac{q}{(a^2+r^2)} \cos\theta \cos\varphi$

$E_y = \sin\varphi \; E_{plane,1,2} = \frac{2}{4\pi\varepsilon_0} \frac{q}{(a^2+r^2)} \cos\theta \sin\varphi$

$E_z = 0.$

$B_x = 0, \quad B_y = B_z = 0, \quad \vec{S} = \vec{0}.$

b. $\vec{da} = (0, 0, -r\,dr\,d\varphi)$

c $(\overleftrightarrow{T} \cdot \vec{da})_z = -\varepsilon_0 \left(E_z E_z - \frac{1}{2} E^2\right) r\,dr\,d\varphi$

$E^2 = \left(\frac{q}{2\pi\varepsilon_0}\right)^2 \frac{r^2}{(a^2+r^2)^3}$

$F_z = \cancel{2}\pi \frac{1}{\cancel{2}} \varepsilon_0 \left(\frac{q^2}{2\pi\varepsilon_0}\right)^2 \int_0^\infty \frac{r^3}{(r^2+a^2)^3} dr$

$= \frac{\pi\varepsilon_0 q^2}{4\pi^2\varepsilon_0^2} \frac{1}{4a^2} = \frac{q^2}{4\pi\varepsilon_0} \left(\frac{1}{2a}\right)^2$

5 a $E = V/d$

$= 300 / 0.02 = 1{,}5\ 10^4\ V/m$

b $E = \dfrac{\sigma}{\varepsilon_0} = \dfrac{Q/A}{\varepsilon_0}$

$Q = E\, A\, \varepsilon_0$

$= 1{,}5.\ 10^4 \times 0{,}2 \times 0{,}1 \times 8{,}9\ 10^{-12}$

$= 2.67.\ 10^{-9}\ C$

$\dfrac{Q}{e} = \dfrac{2.67.\ 10^{-9}\ C}{1{,}6\ 10^{-19}\ C} = 1.7.\ 10^{10}$ electrons

c moving EW; only this length contracted

EW $L = \dfrac{L_0}{\gamma} = \dfrac{0{,}2}{(1-(0{,}6)^2)^{-1/2}} = 0.16\ m$

NS $L = L_0 = 0{,}10\ m$

Top-Down $L = L_0 = 0{,}02\ m$

d Q = conserved $Q = Q_0 = 1.7.10^{10}$ electrons

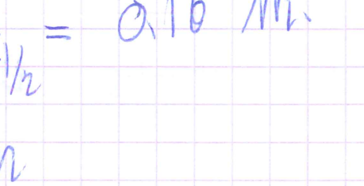
e $E'_\perp = \gamma\, E_{0\perp} = 1{,}25\, E_{0\perp} = 1.875\ 10^4\ V/m$

f $|B'| = \dfrac{v}{c^2} E' = \dfrac{v}{c}\dfrac{1}{c} E' = \dfrac{0.6 \times 1.875\ 10^4}{3.10^8}$

$= 3.75\ 10^{-5}\ T.$

g. Reference frame moving east is same as plates moving west

$\vec{B}$ points N because $\vec{v} \times \vec{B}$ W



h no change $1{,}5\ 10^4\ V/m$ $E'_{\parallel} = E_{0\parallel}$

6 a. Two waves

$\cos(kx + \omega t)$ reflected

$\cos(kx - \omega t)$ incoming.

amplitudes are same for both

Polarization direction is $\hat{x}$.

$\omega / k = c$

b $B = \frac{E_0}{c}\left[\cos(kz - \omega t) + \cos(kz + \omega t)\right]$

$\vec{E} \times \vec{B}$ direction of propagation

thus $\vec{B} = +B_0 \hat{y}$ for $-\omega t$ and $+B_0 \hat{y}$ for $+\omega t$

c $\frac{1}{\mu_0}\vec{B} = \vec{K} \times \hat{n}$

$\frac{1}{\mu_0}\frac{E_0}{c}\left[\cos(\cancel{kz} - \omega t) + \cos(\cancel{kz} + \omega t)\right]\hat{y}$

$\vec{K} = \frac{2E_0}{\mu_0 c}\cos(\omega t)\,\hat{x}$.

d. $f = \langle \vec{K} \times \vec{B} \rangle = \frac{2E_0}{\mu_0 c}\frac{E_0}{c}\langle \cos^2 \rangle$

$= \varepsilon_0 E_0^2$

6 b Alternative using M4:

$$\vec{\partial} \times \vec{B} = \mu_0 \varepsilon_0 \frac{\partial \vec{E}}{\partial t}$$

$$\frac{\partial \vec{E}}{\partial t} = \omega E_0 \hat{x} \left[\sin(kz - \omega t) + \sin(kx + \omega t) \right]$$

$\vec{B} \perp \vec{E}$ and $\vec{B} \perp \hat{k} \Rightarrow \vec{B}$ has only y compon

$$\vec{\partial} \times \vec{B} = \left(\cancel{\frac{\partial B_z}{\partial y}} - \frac{\partial B_y}{\partial z} \right)\hat{x} + \left(\cancel{\frac{\partial B_x}{\partial z}} - \cancel{\frac{\partial B_z}{\partial x}} \right)\hat{y} + \left(\cancel{\frac{\partial B_y}{\partial x}} - \cancel{\frac{\partial B_x}{\partial y}} \right)\hat{z}$$

$$= - \frac{\partial B_y}{\partial z} \hat{x}$$

$$\mu_0 \varepsilon_0 \omega E_0 \left[\sin(kz - \omega t) + \sin(kx + \omega t) \right] = - \frac{\partial B_y}{\partial z}$$

Thus $B_y = -\mu_0 \varepsilon_0 \omega E_0 \int \left[\sin(kx - \omega t) + \sin(kx + \omega t) \right] dz$

$$B_y = + \mu_0 \varepsilon_0 \frac{\omega}{k} E_0 \left[\cos(kx - \omega t) + \cos(kx + \omega t) \right]$$

$$= \frac{1}{c^2} c E_0 \left[\quad \right] =$$

$$= \frac{E_0}{c} \left[\cos(kx - \omega t) + \cos(kx + \omega t) \right]$$

y - direction and in phase with $\vec{E}$